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TIME VARIATION OF DISCHARGE IN INDIVIDUAL OVERTOPPING WAVES

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ABSTRACT

This paper describes additional analysis of wave overtopping measurements acquired during small-scale physical model tests that were part of the *FlowDike* experiments. Previously, time series of instantaneous discharge were calculated as the product of the flow velocity and depth at the seaward crest edge of small-scale dikes having seaward slopes of either 1V:3H or 1V:6H. Individual overtopping waves were identified by a two-step process that resulted in a highly reliable data set of 5,799 individual waves. In this paper, the two-parameter Weibull probability density function is adopted to represent the shape of overtopping wave time-varying instantaneous discharge. Values of the Weibull scale factors, *a*, and shape factors, *b*, are obtained through nonlinear best-fitting of the Weibull equation to all 5,799 waves. Similar best-fits were performed for the Rayleigh version of the equation when $b = 2$. An empirical equation is developed for scale factor, *a*; but similar success was not achieved for the best-fit values of shape factor, *b*. Predictions of time-varying discharge made using the one-parameter Rayleigh equation are assessed in terms of the root-mean-square errors between predictions and measurements. The empirical formulation for time-varying discharge can be used to estimate the cumulative excess work in overtopping waves.

KEWORDS: Wave overtopping, time-varying discharge, cumulative overtopping volume, prediction, Weibull equation.

1 INTRODUCTION

The capability of grass-covered earthen dikes and levees to withstand tolerable rates of wave overtopping is related almost entirely to the resiliency of the grass/soil system. Hydrodynamic forces exerted on the structure crest and landward-side slope grass/soil system are unsteady with significant variation of the instantaneous discharge during each overtopping wave. Maximum flow discharge can be several times greater than the average discharge of the overtopping event.

Engineers have developed full-scale wave overtopping simulators to investigate resiliency of grass and other slope protection products (Van der Meer *et al*. 2006; Van der Meer *et al*. 2008). Overtopping simulators attempt to replicate the wave overtopping hydraulics by releasing volumes of water in such a manner that key parameters in each wave volume are correctly reproduced. Even better simulations could be performed if the overtopping simulator water release approximated the actual time variation in instantaneous discharge as well. This improvement will lead to robust design guidance and greater resiliency for grass-covered dikes and levees.

A valuable data set pertaining to irregular wave overtopping was obtained during the extensive small-scale *FlowDike* experiments conducted at the Danish Hydraulic Institute in Hørsholm, Denmark. These experiments examined wave run-up and overtopping on smooth dikes having planar seaward-side slopes of 1-on-3 and 1-on-6 (vertical-on-horizontal). Complete descriptions of both *FlowDike* experiments are given by Lorke *et al*. (2009) and Lorke *et al*. (2010).

Hughes (2015a; 2015b) analyzed a subset of the *FlowDike* data measured for normally-incident irregular waves with no added current or wind, and he developed a well-vetted data set of 5,799 individual overtopping waves. Individual overtopping wave volumes and the associated key parameters were determined at the seaward edge of the dike crest for incident wave conditions impinging on the 1-on-3 and 1-on-6 planar seaward-side dike slopes. Time series of instantaneous discharge were calculated as the product of measured velocity and flow thickness. Identification of individual wave volumes was achieved using a two-step "supervised" method that combined the best features of automated wave volume determination along with manual error correction. Correlations between the individual wave volumes and the associated key hydraulic parameters resulted in new empirical equations for maximum velocity, maximum flow thickness, maximum discharge per unit crest length, and the overtopping duration occurring in overtopping volumes.

This paper examines the shape of the time-varying instantaneous discharge per unit width [$q(t)$] in an overtopping wave, as well as the time-varying cumulative overtopping volume per unit width [$V(t)$] using the previously identified 5,799 waves. These time-varying phenomena are approximated by theoretical equations, and the equations are fitted to the discharge and cumulative volume data using a non-linear best-fit procedure. The best-fit variables are then correlated to predictable wave overtopping parameters in an attempt to develop empirical expressions for $q(t)$ and $V(t)$. The new predictive equations are evaluated in terms of the root-mean-square error when compared to measured data, and an application to the cumulative excess work methodology is discussed. An expanded version of this research is given in a technical report (Hughes and Thornton 2015).

2 PROPOSED TIME-VARYING EQUATIONS

The time variation of instantaneous discharge in individual overtopping waves typically features a rapid increase in discharge to a peak value, followed by a slower decline in discharge. This distribution of discharge in time resembles the form of the familiar Weibull distribution equation as shown by Equation (1) below. Equation (2) is the associated cumulative distribution of overtopping volume obtained by integrating Equation (1) with respect to time.

$$q(t) = \frac{V_T\, b}{a} \left(\frac{t}{a}\right)^{(b-1)} \exp\left[-\left(\frac{t}{a}\right)^{b}\right] \qquad \text{Discharge Distribution} \qquad (1)$$

$$V(t) = V_T \left\{1 - \exp\left[-\left(\frac{t}{a}\right)^{b}\right]\right\} \qquad \text{Cumulative Overtopping Volume} \qquad (2)$$

In Equations (1) and (2), $q(t)$ is instantaneous discharge per unit dike width, $V(t)$ is cumulative overtopping volume per unit dike width, V_T is total wave volume per unit width, t is time, a is the distribution scale factor, and b is the distribution shape factor. The total wave volume per unit width, V_T, is included so the cumulative wave volume is equal to total wave volume when t approaches infinity, i.e., $V(t \rightarrow \infty) = V_T$. Equations (1) and (2) revert to the familiar Rayleigh distribution equations when the shape factor, $b = 2$.

A few characteristic features of the overtopping wave time-varying discharge distribution and time-varying cumulative overtopping volume being represented by the Weibull equations can be easily derived. The time at the occurrence of the maximum (or peak) discharge is found by differentiating Equation (1) and setting the result equal to zero. After some algebraic manipulation, the theoretical time at maximum discharge, t_{maxT}, is given by

$$t_{maxT} = a \left(\frac{b-1}{b}\right)^{1/b} \qquad (3)$$

where the upper-case "T" is used in the subscript (here and in subsequent equations) to denote "*theoretical*." Substituting t_{maxT} from Equation (3) for t in Equation (1) yields an expression for the theoretical maximum discharge in an overtopping wave, i.e.,

$$q_{maxT} = \frac{V_T\, b}{a} \left(\frac{b-1}{b}\right)^{\frac{b-1}{b}} \exp\left[-\left(\frac{b-1}{b}\right)\right] \qquad (4)$$

An approximation of the total overtopping duration can be estimated from the cumulative overtopping volume Equation (2) by calculating the time it takes for the cumulative volume to reach (say) 99% of the total volume. In other words,

$$V(t = T_{oT}) \approx 0.99\, V_T = V_T \left\{1 - \exp\left[-\left(\frac{T_{oT}}{a}\right)^{b}\right]\right\} \qquad (5)$$

Of course, it would be reasonable to select some other arbitrary total volume percentage (say 95% or 99.9%) to define the overtopping duration. Solving Equation (5) for T_{oT} , and noting that mean discharge is defined as total overtopping volume divided by overtopping duration, yields an approximate equation for theoretical mean overtopping discharge in an individual overtopping wave given by

$$q_{meanT} = \frac{V_T}{T_{oT}} = \frac{V_T}{a\ [-\ln(0.01)]^{1/b}} \qquad (6)$$

The value of mean discharge given by Equation (6) is strictly for a single individual overtopping wave, and it should not be confused with the average overtopping discharge of the storm event. Equations (3) – (6) reduce to the Rayleigh equation versions when the shape factor, $b = 2$.

3 BEST-FITS TO MEASUREMENTS

Non-linear least-squares best-fits of the proposed Weibull instantaneous discharge distribution (Equation 1) and the corresponding Rayleigh version of Equation (1) were performed using as the target the time-varying instantaneous discharge measurements for all 5,799 individual overtopping waves. Similarly, best-fits of the proposed Weibull cumulative volume distribution (Equation 2) and the Rayleigh version of Equation (2) were performed using the time variation of cumulative overtopping volume that was calculated as the cumulative summation of instantaneous discharge at each point multiplied by the incremental time between measurement points. The best-fit analysis gave a total of 4 x 5,799 = 23,196 least-squares best-fit results for the FlowDike data set.

An indication of the goodness-of-fit was judged by the square of the correlation coefficient (sometimes referred to as the coefficient of determination) defined as

$$r^2 = \frac{\sum_{i=1}^{N}(q_i - \bar{q}_m)^2}{\sum_{i=1}^{N}\left(q_i - q_{m,i}\right)^2 + \sum_{i=1}^{N}(q_i - \bar{q}_m)^2} \quad or \quad r^2 = \frac{\sum_{i=1}^{N}(V_i - \bar{V}_m)^2}{\sum_{i=1}^{N}\left(V_i - V_{m,i}\right)^2 + \sum_{i=1}^{N}(V_i - \bar{V}_m)^2} \tag{7}$$

where

q_i = best-fit value of instantaneous discharge at time increment i

$q_{m,i}$ = measured instantaneous discharge at time increment i

$\bar{q}_m$ = mean of measured instantaneous discharges in wave

V_i = best-fit value of cumulative wave volume at time increment i

$V_{m,i}$ = measured cumulative wave volume at time increment i

$\bar{V}_m$ = mean of measured cumulative volume in wave

N = total number of increments in overtopping wave

Values of the coefficient of determination, r^2, as a function of individual wave volume for all 5,799 wave volumes are shown in Figure 1 for both the Rayleigh and Weibull versions of the equations. The left-hand plot compares r^2 values for the time-varying discharge (Equation 1), and the right-hand plot compares r^2 values for the time-varying cumulative overtopping volume (Equation 2). In many, but not all, cases the Weibull equations provided a better fit to the measured data, and the equation for cumulative overtopping volume gave the best fits. Table 1 lists the means of all 5,799 r^2 values.

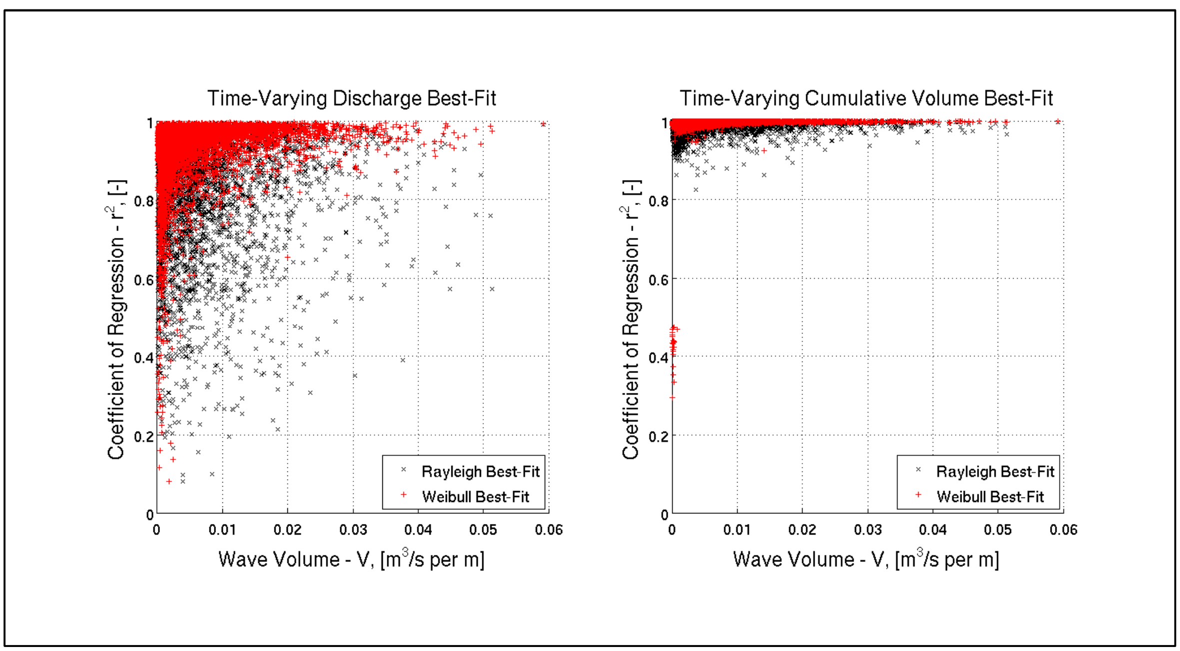

Figure 1. Comparison of r^2 values for best-fits using Rayleigh and Weibull equations.

Table 1. Mean values of r^2 for best-fit equations.

Discharge Distribution		Cumulative Volume Distribution	
Rayleigh Eqn.	Weibull Eqn.	Rayleigh Eqn.	Weibull Eqn.
0.860	0.932	0.988	0.993

Values of the scale factor, *a*, obtained from the best-fits of both the Weibull and Rayleigh versions of the time-varying discharge Equation (1) are compared in the left-hand plot of Figure 2 to the corresponding scale factors obtained from best-fits of the time-varying cumulative overtopping volume Equation (2). The blue markers are from the Rayleigh equation versions, and the red markers represent the Weibull equations. The solid line is the line of equivalence. There is fairly good correspondence between the discharge equation and the cumulative volume equation best-fit values for scale factor. The best comparison was found between the Rayleigh and Weibull versions of the time-varying cumulative volume Equation (2) (Hughes and Thornton, 2015). This implies that similar values of scale factor, *a*, can be obtained from either the Rayleigh or Weibull versions of Equation (2). Fortunately, this facilitates the reasonable scale factor correlation shown in Section 4.

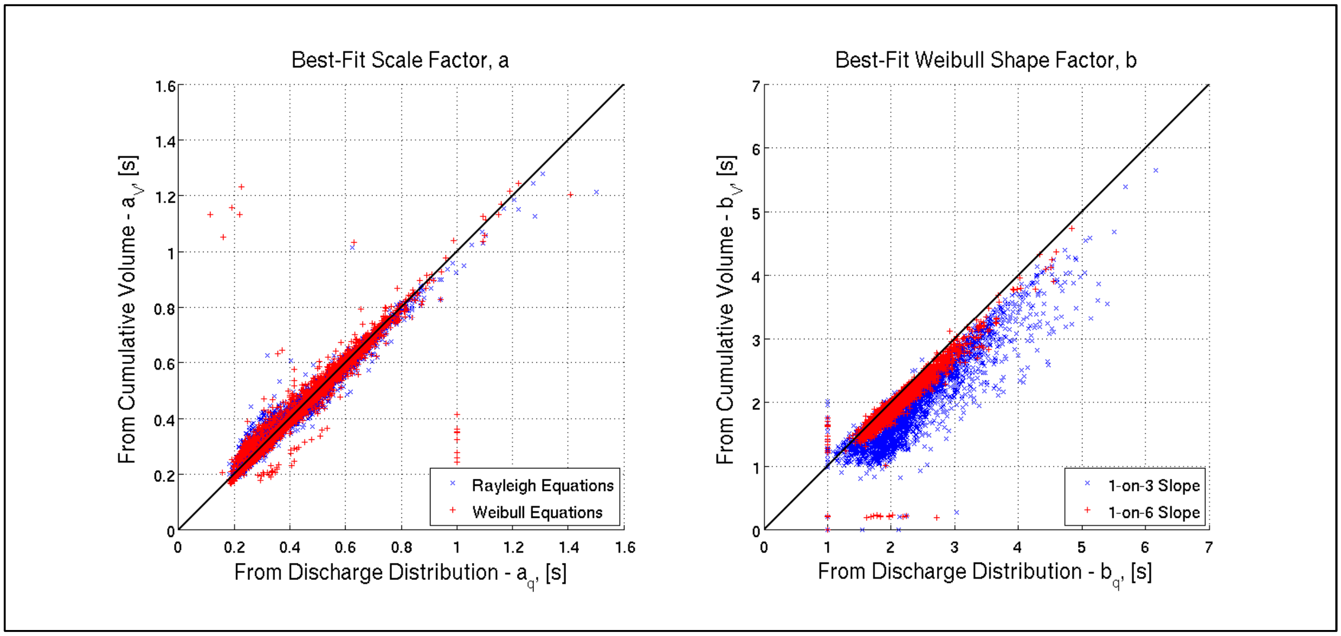

Figure 2. Best-fit scale factors, *a*, and shape factors, *b*.

The Weibull shape factor, *b*, obtained from the best-fit of Equation (1) is compared in the right-hand plot of Figure 2 to the corresponding shape factor from the best-fit of Equation (2). Data from the 1-on-3 slope are shown by blue markers, and data from the 1-on-6 slope are shown by red markers. The solid line is the line of equivalence. Generally, larger shape factors result from the best-fits of Equation (1) to the data representing the time-varying instantaneous discharges, and this illustrates a definite bias depending on which equation was used for the best-fit. This difference in best-fit shape factors complicated discovering an empirical representation for shape factor.

In summary, it has been shown that the two-parameter Weibull equations for the time variation of instantaneous discharge and cumulative overtopping volume provide reasonably good approximations of measured wave overtopping data obtained during the *FlowDike* experiments. Furthermore, the one-parameter Rayleigh versions of the same equations provided best-fits that were not much worse than the two-parameter Weibull equations.

4 CORRELATION FOR SCALE FACTOR, *a*

The theoretical equations for time-varying discharge and cumulative volume in overtopping waves (Equations 1 – 6) require appropriate values of the scale factor, *a*, and the shape factor, *b*, expressed in terms of predictable key parameters of individual overtopping waves. The approach taken was to consider two equations derived from the theoretical time-varying Weibull equations: the theoretical maximum discharge, q_{max}, (Equation 4), and the theoretical mean discharge, q_{mean}, (Equation 6). Assuming that q_{max} and q_{mean} can be appropriated in terms of overtopping wave parameters, Equations (4) and (6) are two equations with two unknowns, *a* and *b*. Both equations are transcendental forms in terms of the unknowns, so a simple algebraic solution is not possible. An appropriate solution for the Weibull version of the equations is still being investigated, but preliminary results are presented in this paper based on the slightly less accurate Rayleigh equations.

The best-fit Weibull scale factors, a, were shown in Figure 2 to be quite similar irrespective of which equation was used for the best-fit (time-varying discharge or cumulative volume) and whether or not the Raleigh or Weibull version was applied. This observation simplifies the correlation for the best-fit shape factor. Rearranging the Rayleigh version (b = 2) of Equation (6) for mean overtopping discharge yields an equation for scale factor in terms of individual overtopping wave volume (V_T) and theoretical mean overtopping discharge (q_{meanT}), i.e.,

$$a_T = \frac{V_T}{q_{meanT}\ [-\ln(0.01)]^{1/2}} = \frac{V_T}{q_{meanT}\ (2.146)} \tag{8}$$

where a_T represents the theoretical shape factor associated with the Rayleigh version of the cumulative volume equation.

An analysis was performed to determine an appropriate correlation between measured and theoretical estimates of the mean discharge per unit width. The left-hand plot of Figure 3 shows theoretical discharge (q_{meanT}) determined from the Weibull version of Equation (6) using best-fit parameters versus the measured parameter [q_{mean} (tan α)$^{1/2}$]. The solid black line on the left-hand plot of Figure 3 is the linear best-fit passing through the origin given by the equation

$$q_{meanT} = 3.27\ q_{mean}\ \sqrt{\tan\alpha} \tag{9}$$

where q_{meanT} is the theoretical mean discharge per unit width from Equation (6), q_{mean} is measured mean discharge per unit width; and α is the angle of seaward dike slope relative to horizontal. This best-fit equation had a correlation coefficient of C_c = 0.982, a coefficient of determination of r^2 = 0.964, and a root-mean-square error of e_{RMS} = 0.0013 m^3/s per m.

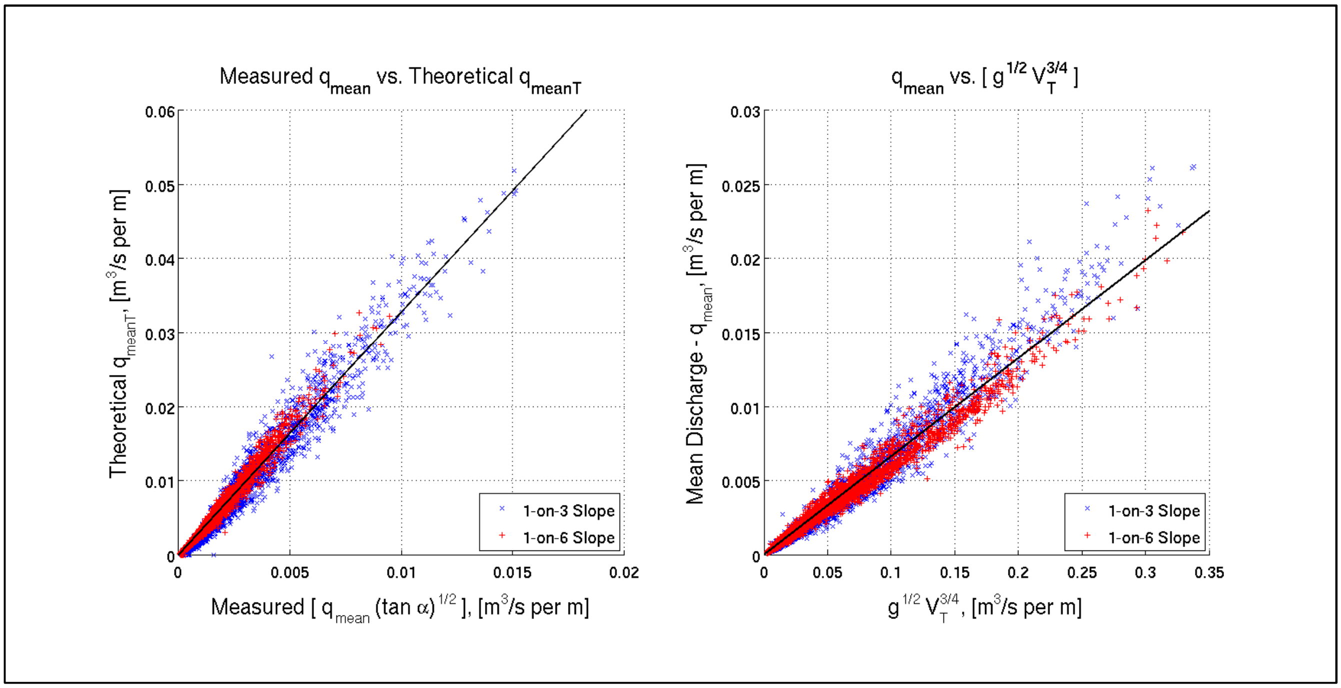

Figure 3. Correlation for theoretical mean discharge, q_{meanT}.

Calculated values of mean discharge per unit width from all 5,799 individual overtopping waves were then examined to determine a reasonable correlation in terms of overtopping wave parameters. It appeared that mean discharge was chiefly a function of wave volume only, and dimensional analysis suggested that wave volume should be raised to the ¾-power for dimensional consistency. The right-hand plot of Figure 3 shows calculated mean discharge plotted versus the parameter $\sqrt{g}\ V_T^{3/4}$. Gravity was included to balance the dimensions. A linear regression provided a best-fit, shown by the black line in the right-hand plot of Figure 3, given by the equation

$$q_{mean} = 0.066\ \sqrt{g}\ V_T^{3/4} \tag{10}$$

where g is gravitational acceleration and V_T is the total volume in an individual overtopping wave. This best-fit equation had a correlation coefficient of C_c = 0.978, a coefficient of determination of r^2 = 0.956, and a root-mean-square error of e_{RMS} = 0.0008 m^3/s per m.

Substituting Equation (10) for measured mean discharge (q_{mean}) into Equation (9), and then substituting the result into Equation (8) yields a semi-empirical expression for theoretical scale factor, a_T, in terms of individual overtopping wave volume and dike seaward-side slope angle, i.e.,

$$a_T = \frac{2.16\, {V_T}^{1/4}}{\sqrt{g \tan \alpha}} \tag{11}$$

Figure 4 compares the semi-empirical scale factor prediction given by Equation (11) to the best-fit scale factor determined from the Rayleigh version of the cumulative overtopping volume equation. Blue markers indicate the 1-on-3 slope data, and red markers denote 1-on-6 slope data. The solid black line in Figure 4 is the best-fit linear regression given by the equation

$$a_V = 1.13\, a_T \tag{12}$$

This best-fit equation had a correlation coefficient of C_c = 0.692, a coefficient of determination of r^2 = 0.480, and a root-mean-square error of e_{RMS} = 0.107 s. Naturally, the correlation between theoretical and best-fit scale factors exhibits scatter, but the general trend is seen to be representative of the 5,799 measured individual overtopping waves. The scatter represents the combined uncertainty present in both the data itself and in the empirical representation of the measured mean discharge (q_{mean}).

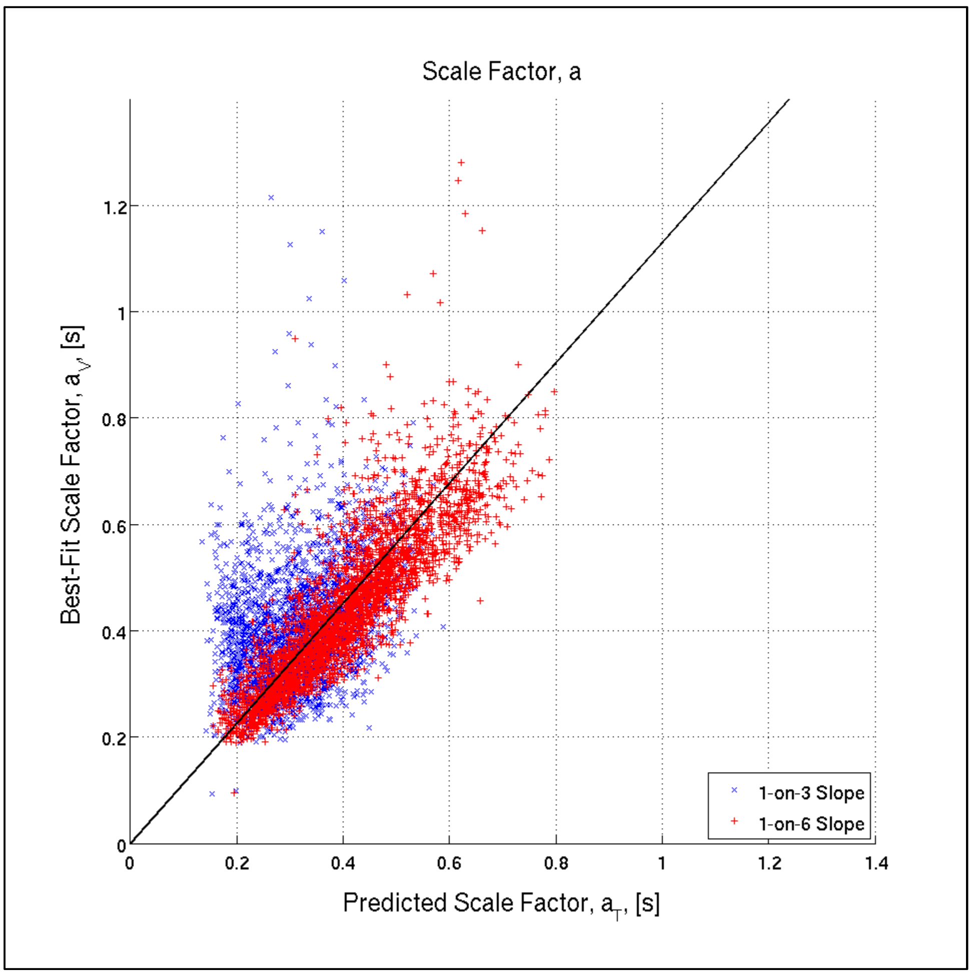

Figure 4. Correlation for scale factor, *a*, from Rayleigh cumulative volume equation.

Substituting Equation (11) into Equation (12), and assuming that a_V is a reasonable representation of scale factor, a, determined by any of the equations, a final estimation of the Weibull scale factor is given by

$$a = \frac{2.44\, {V_T}^{1/4}}{\sqrt{g \tan \alpha}} \tag{13}$$

5 TIME-VARYING DISCHARGE PREDICTION ASSESSMENT

The root-mean-square (RMS) errors associated with the best-fits to measured data and predictions to measured data were calculated for the time-varying discharge and the time-varying cumulative overtopping volume as

$$E_{RMS} = \sqrt{\frac{\sum_{i=1}^{N}\left(q_i - q_{m,i}\right)^2}{N}} \quad or \quad E_{RMS} = \sqrt{\frac{\sum_{i=1}^{N}\left(V_i - V_{m,i}\right)^2}{N}} \tag{14}$$

where

q_i = best-fit (or predicted) value of instantaneous discharge at time increment i

$q_{m,i}$ = measured instantaneous discharge at time increment i

V_i = best-fit (or predicted value) of cumulative wave volume at time increment i

$V_{m,i}$ = measured cumulative wave volume at time increment i

N = total number of increments in overtopping wave

The average relative RMS errors for all four best-fit comparisons and for the two Rayleigh prediction comparisons are given in Table 2. The predictions, of course, had larger relative RMS errors; but the error magnitudes are not excessive.

Table 2. Average relative root-mean-square errors of best-tits and predictions.

Method	Mean E_{rms}/q_{max}		Mean E_{rms}/V_T	
	Rayleigh Eqn.	Weibull Eqn.	Rayleigh Eqn.	Weibull Eqn.
Best-Fit	0.104	0.075	0.032	0.018
Prediction	0.157	-	0.079	-

Additionally, predictions were made for the magnitude and time of occurrence of the maximum discharge using Equations (4) and (3), respectively. In these equations, the shape factor was set at b =2 for the Rayleigh approximation, and the scale factors, a, were calculated using Equation (13). The estimation of maximum discharge was quite reasonable as shown on the left-hand plot of Figure 5.

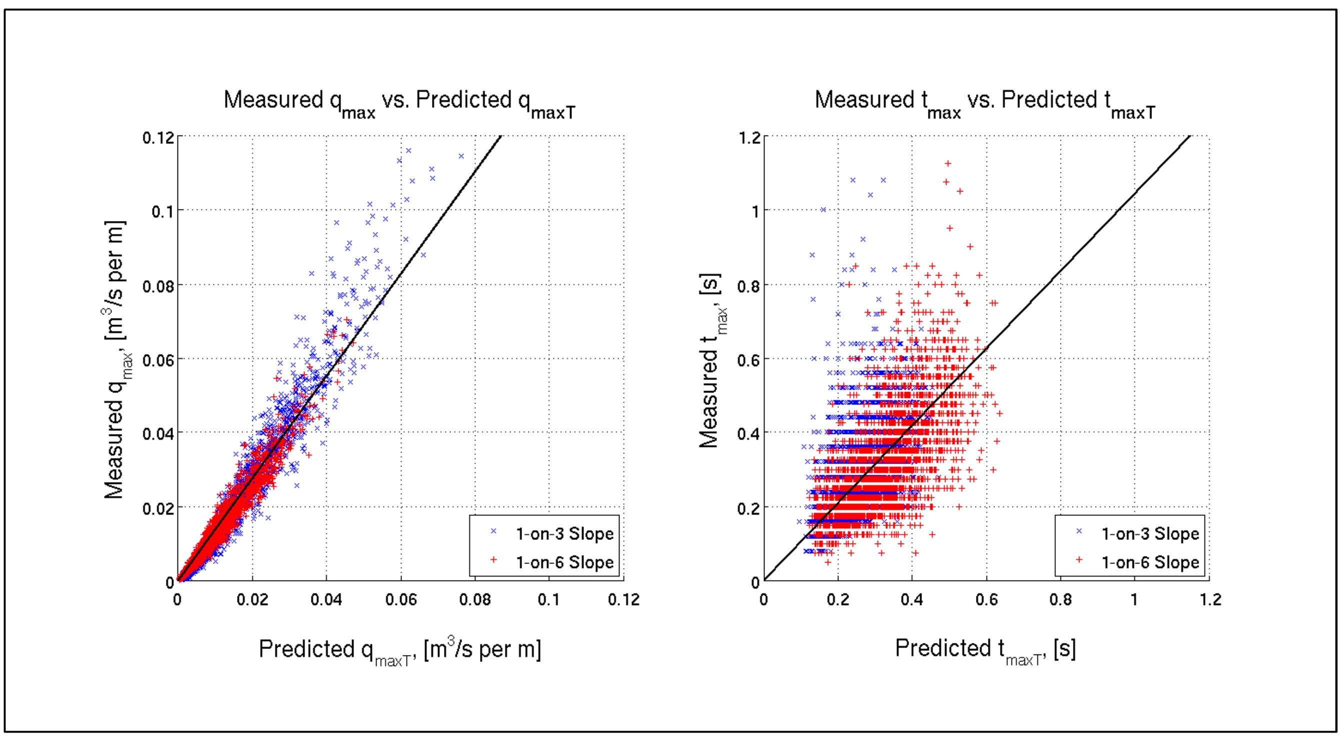

Figure 5. Predicted magnitude and time of maximum discharge in overtopping waves.

The solid line on the left-hand plot of Figure 5 is a best-fit linear regression through the origin given by the equation

$$q_{max} = 1.38\, q_{maxT} = 0.484\sqrt{g \tan\alpha}\;\; V_T^{3/4} \tag{15}$$

This best-fit equation had a correlation coefficient of C_c = 0.975, a coefficient of determination of r^2 = 0.951, and a root-mean-square error of e_{RMS} = 0.0035 m^3/s per m.

Prediction of the time of maximum discharge was not as good as seen on the right-hand plot of Figure 5. The solid line is a linear best-fit through the origin given by the equation

$$t_{max} = 1.044\, t_{maxT} = 0.30\, \frac{\sqrt{g \tan\alpha}}{V_T^{1/4}} \tag{16}$$

This best-fit equation had a relatively poor correlation coefficient of C_c = 0.598, a coefficient of determination of r^2 = 0.357, and a root-mean-square error of e_{RMS} = 0.11 s.

Figure 6 shows four examples of the measured time-varying discharge compared to best-fits of the Weibull discharge distribution (solid blue lines) and the Rayleigh equation predictions (dashed red lines). The plots are ordered by best-fits having progressively better r^2-values. The best-fit curves are Equation (1) with best-fit values of scale and shape factors. The predictions were made using Equation (1) with b = 2 and the scale factor, a, given by Equation (13). Note that maximum (or peak) discharge of the theoretical discharge distribution is typically less than the measured maximum peak (as verified by Equation 15). Obviously, the Weibull best-fits more closely match the measurements, but the Rayleigh prediction presented in this paper is entirely reasonable…except perhaps in the lower left-hand plot of Figure 6.

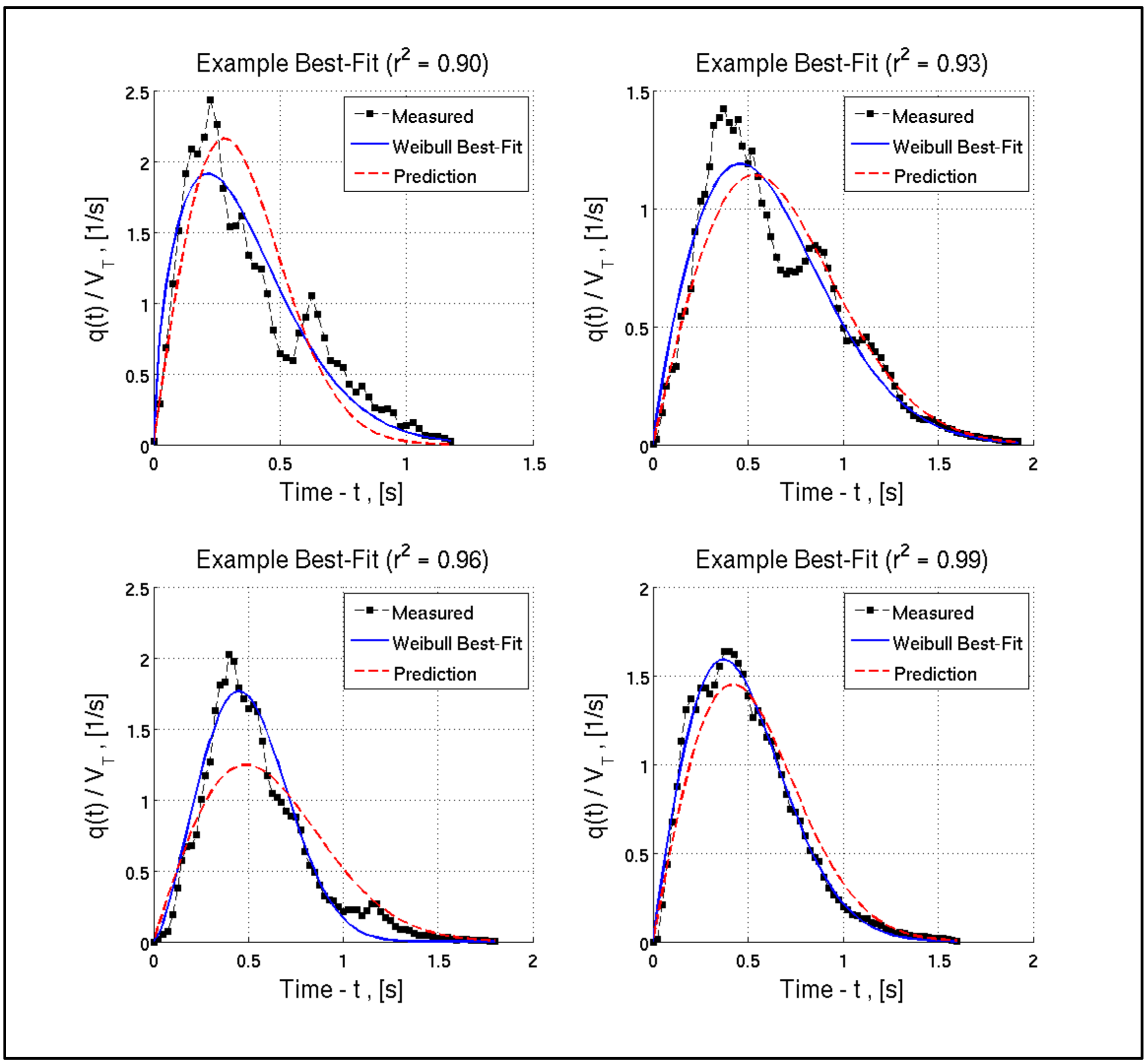

Figure 6. Comparison of measured, best-fit, and Rayleigh equation prediction of time-varying discharge.

6 APPLICATION TO CUMULATIVE EXCESS WORK METHODOLOGY

Dean *et al*. (2010) examined whether flow velocity (u), shear stress ($\propto u^2$), or work ($\propto u^3$) above a given threshold was the best parameter for relating design nomograms of grass stability derived from steady overtopping measurements to the case of unsteady wave overtopping. They concluded flow work ($\propto u^3$) above a certain threshold provided the best estimator of erosion, and the concept was named "*erosional equivalence*". Summing the contribution from all the overtopping waves led to the term "*cumulative excess work*" (CEW), which is quite similar to the concept of *hydraulic loading* developed by Van der Meer *et al*. (2010).

Hughes (2011) expanded Dean *et al*.'s concept by showing the excess work in the overtopping wave can represented by the sum of the time-varying discharge above a critical discharge threshold, i.e.,

$$W_E = \int_{T_A}^{T_B} [q(t) - q_c]\, dt = V_E \qquad \text{For } q(t) \geq q_c \tag{17}$$

where W_E is excess flow work in an overtopping wave, q_c is critical threshold discharge, T_A is the time when instantaneous discharge first exceeds critical discharge, T_B is the time when instantaneous discharge again equals critical discharge, and V_E is excess volume in an overtopping wave. In other words, the excess work in an overtopping wave is equivalent to the wave volume above the critical discharge threshold.

Hughes (2011) developed a predictive model for the cumulative excess work by assuming an idealized saw-tooth shape for the time-varying instantaneous discharge. However, the results from this paper provide a more realistic representation of time-varying discharge that can be used in the methodology. The left-hand plot of Figure 7 illustrates cumulative excess work (or volume) using the time-varying discharge predicted by Equation (1) for a wave volume equal to the maximum of the Dutch overtopping simulator ($V_T = 5.5$ m^3/m). The critical discharge, shown by the horizontal red line, is approximately equivalent to a critical velocity of 6 m/s on a 1-on-3 landward-side dike slope. The wave volume per unit width above the critical discharge is the excess volume (or work) that contributes to erosion. The excess volume is equal to 3.41 m^3/m, which is 62% of the total wave volume. The plot on the right-hand side of Figure 7 is a wave that has half the volume ($V_T = 2.75$ m^3/m) of the wave shown on the left-hand plot, and in this case the excess volume contributing to erosion is 0.81 m^3/m, or 29% of the total volume. Further details of the cumulative excess work concept and suggested implementation into a predictive model are given in Hughes (2011).

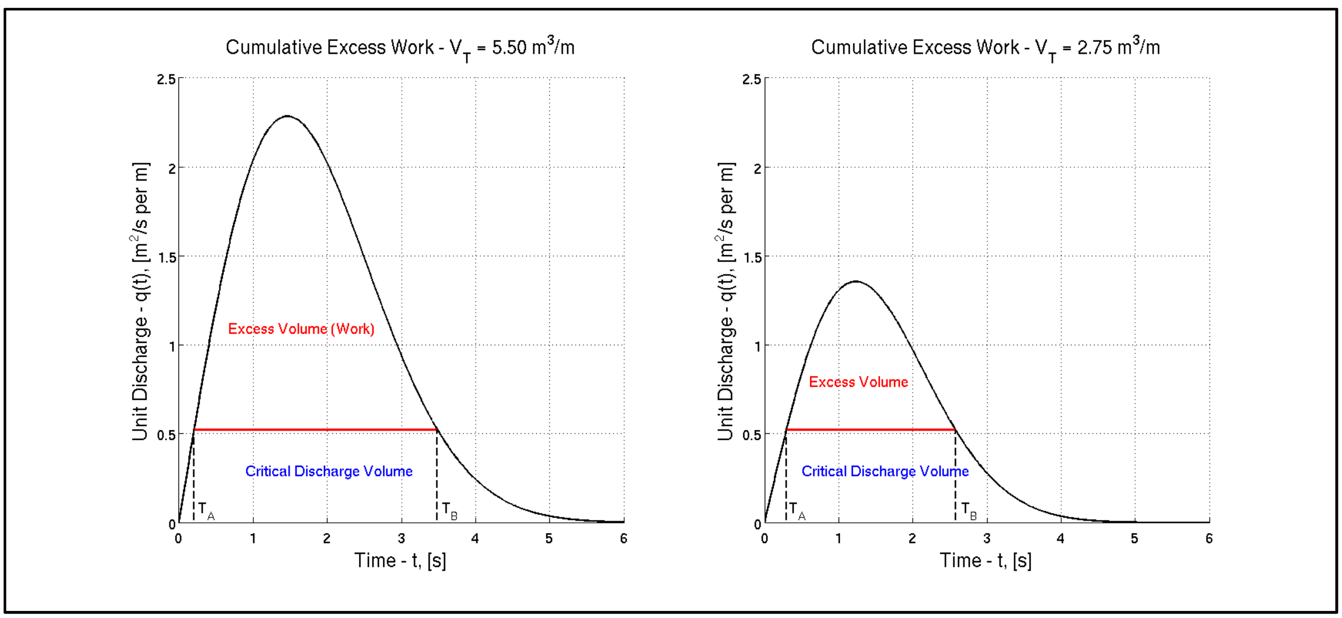

Figure 7. Examples of excess work (volume) above the threshold in overtopping waves.

7 CONCLUSIONS

This study examined previously determined time series of instantaneous discharge of individual overtopping waves measured at the seaward edge of the dike crest for incident wave conditions impinging on either a 1-on-3 or 1-on-6 planar seaward-side dike slope. The shapes of the time-varying discharge and time-varying cumulative volume (per unit width) were well described by the two-parameter Weibull equations, as shown by nonlinear best-fits to measurements from 5,799 overtopping waves. The associated one-parameter Rayleigh equations with shape factor, $b = 2$, did almost as well.

A reasonable empirical equation was determined that related the best-fit values of scale factor, a, to the individual overtopping wave volume and the seaward-side slope of the dike. However, work is still in progress to find a suitable empirical relationship for the best-fit values of shape factor, b. In the meantime, a predictive set of equations using a shape factor of $b = 2$ is recommended (i.e., the Rayleigh versions of Equations 1 – 6).

Assessment of the root-mean-square errors between predictions and measurements indicated the one-parameter Rayleigh versions of the equations provided a reasonable estimate of the time-varying discharge, and an empirical equation provided good predictions of the maximum discharge. The new empirical formulation for time-varying discharge in overtopping waves can be used in the cumulative excess work methodology to assess the erosional resiliency of earthen levees subjected to wave overtopping.

The new equations presented in this paper strictly apply at the seaward edge of the dike crest on dikes having planar seaward-side slopes ranging between 1V-on-3H to 1V-on-6H. These equations may prove useful for additional refinements to the science of full-scale wave overtopping simulation.

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REFERENCES

Dean, R. G., Rosati, J. D., Walton, T. L., and Edge, B. L. 2010. "Erosional equivalences of levees: Steady and intermittent wave overtopping," In: Demirbilek (editor), *Special issue of Ocean Engineering*, Elsevier, Vol 37, pp 104-113.

Hughes, S. A. 2011. "Adaptation of the levee erosional equivalence method for the Hurricane Storm Damage Risk Reduction System (HSDRRS)," ERDC/CHL TR-11-3, U.S. Army Engineer Research and Development Center, Vicksburg, Mississippi, 125pp.

Hughes, S. A. 2015a. "Hydraulic parameters of individual overtopping wave volumes," Technical Report, Engineering Research Center, Colorado State University, Fort Collins, CO, 68pp.

Hughes, S. A. 2015b. "Hydraulic parameters of overtopping wave volumes," , *Proceedings of Coastal Structures 2015,* Boston, Massachusetts, American Society of Civil Engineers, New York, (in press).

Hughes, S. A. and Thornton, C. I. 2015. "Time-varying discharge of individual overtopping wave volumes," Technical Report, Engineering Research Center, Colorado State University, Fort Collins, CO, 56pp.

Lorke, S., Brüning, A., Bornschein, A., Gilli, S., Krüger, N., Schüttrumpf, H., Pohl, R., Spano, M., and Werk, S. 2009. "Influence of wind and current on wave run-up and wave overtopping," Hydralab – FlowDike, Report 2009, 100pp.

Lorke, S., Brüning, A., Van der Meer, J., Schüttrumpf, H., Bornschein, A., Gilli, S., Pohl, R., Spano, M., Řiha, J., Werk, S., and Schlütter, F. 2010. "On the effect of current on wave run-up and wave overtopping," *Proceedings of the 32nd International Conference Coastal Engineering*, Shanghai, China, American Society of Civil Engineers. New York. Available at: https://journals.tdl.org/icce/index.php/icce/article/download/1388/pdf_366.

Van der Meer, J. W., Snijders, W., and Regeling, E. 2006. "The wave overtopping simulator," *Proceedings of the 30th International Coastal Engineering Conference*, American Society of Civil Engineers, Vol 5, pp. 4654-4666.

Van der Meer, J. W., Steendam, G. J., de Raat, G., and Bernardini, P. 2008. "Further developments on the wave overtopping simulator," *Proceedings of the 31st International Conference Coastal Engineering*, American Society of Civil Engineers, Vol 4, pp. 2957-2969.

Van der Meer, J. W., Hardeman, B., Steendam, G. J., Schüttrumpf, H., and Verheij, H. 2010. "Flow depths and velocities at crest and inner slope of a dike, in theory and with the wave overtopping simulator", *Proceedings of the 32nd International Conference Coastal Engineering*, Shanghai, China, American Society of Civil Engineers. New York. Available at: https://journals.tdl.org/icce/index.php/icce/article/download/1239/pdf_260.